

Tsinghua-Princeton-CI Summer School
July 14-20, 2019

Structure and Dynamics of Combustion Waves in Premixed Gases

Paul Clavin

Aix-Marseille Université
ECM & CNRS (IRPHE)

Lecture IX Turbulent flames

Lecture 9 : **Turbulent flames**

9-1. Introduction

9-2. Turbulent diffusion

Einstein-Taylor's diffusion coefficient

Rough model of turbulent transport

Well-stirred flame regime

9-3. Strongly corrugated flammelets regime

Kolmogorov's cascade

Gibson's scale

Elements of fractal geometry

Self similarity of strongly corrugated flames

Co-variant laws

9-4. Turbulent combustion noise

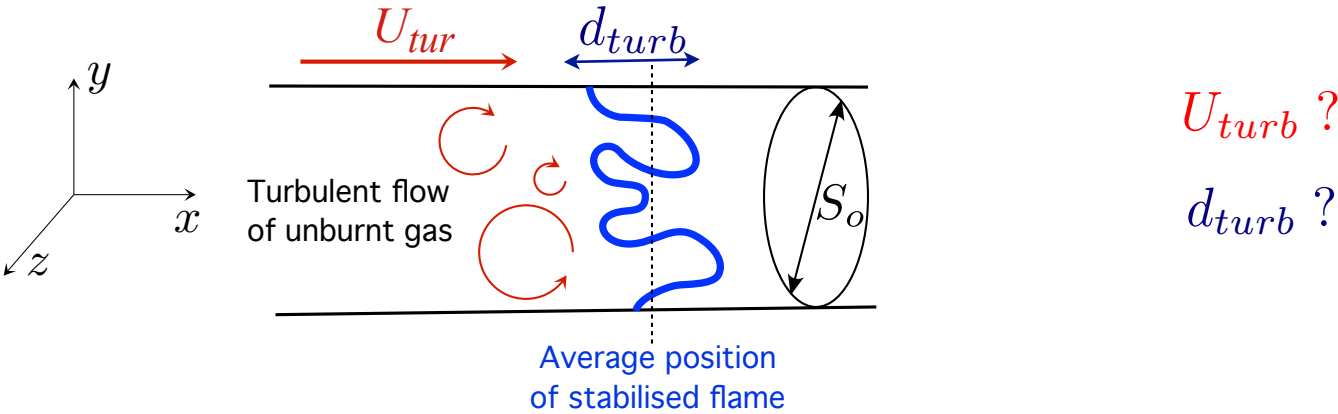
Monopolar sound emission

Sound generated by a turbulent flame

Blow torch noise

IX-1) Introduction

The problem of premixed flames in a turbulent flow is still widely open



Experiments are difficult. Experimental data are very scattered

The simplest model has no known solution (Nonlinear stochastic equation)

Reaction-diffusion wave in a turbulent flow (no gas expansion)

$\partial\theta/\partial t + \mathbf{v}(\mathbf{r}, t) \cdot \nabla\theta - D_T \Delta\theta = \omega'(\theta)/\tau_{rb}.$
prescribed turbulent flow (stochastic field)

Same model in the wrinkled flame regime ($l_{tur} \gg d_L, \tau_{tur} \gg \tau_L \Rightarrow U_n = U_L$)

eq. flame surface $G(\mathbf{r}, t) = G_0$ $\partial G/\partial t + (\mathbf{dr}/dt) \cdot \nabla G = 0$ $\mathbf{dr}/dt = \mathbf{v}(\mathbf{r}, t) - U_n \mathbf{n}$ $\mathbf{n} = \nabla G/|\nabla G|$

stochastic eikonal eq.
 $\partial G/\partial t + \mathbf{v}(\mathbf{r}, t) \cdot \nabla G = U_n |\nabla G|$

$\mathbf{v} = (u, w_y, w_z)$
 $x = \alpha(y, z, t)$
 $G - G_0 = x - \alpha(y, z, t)$ $\partial\alpha/\partial t - u(\mathbf{r}_f, t) + \mathbf{w}(\mathbf{r}_f, t) \cdot \nabla_{yz} \alpha = U_{tur} - U_n \sqrt{1 + |\nabla_{yz} \alpha|^2}$

$\langle S \rangle = \iint dx dy \left\langle \sqrt{1 + |\nabla_{yz} \alpha|^2} \right\rangle$ $U_{tur}/U_L = \left\langle \sqrt{1 + |\nabla_{yz} \alpha|^2} \right\rangle$ $U_{tur} S_o = U_L \langle S \rangle$

The very existence of $\langle S \rangle$ and d_{tur} is questionable

$|\mathbf{v}| \ll U_L : \quad U_{tur}/U_L \approx 1 + (|\mathbf{v}|/U_L)^2$
(Shchelkin 1943, Clavin Williams 1979)

$|\mathbf{v}| \gg U_L : \quad U_{tur} \approx |\mathbf{v}|$
(Damköler 1940)

Bending effect
modification to the laminar flame struture

IX-2) Turbulent diffusion

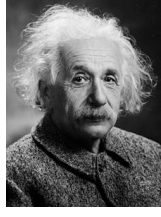
Taylor's diffusion coefficient (analogy with Einstein random walk for molecular diffusion)

1-D for simplicity: $dx/dt = v(t), \quad x(t) = \int_0^t v(t')dt'$
↙ ensemble average
↖ stochastic Lagrangian velocity

$\langle x^2(t) \rangle = \int_0^t dt' \int_0^t dt'' \langle v(t')v(t'') \rangle \qquad \langle x^2(t) \rangle = 2 \int_0^t dt' \int_0^{t'} d\tau \langle v(t')v(t' - \tau) \rangle$



G.I Taylor 1922



Einstein 1905

turbulence: homogeneous in time $\langle v(t)v(t - \tau) \rangle = \langle v^2 \rangle g(\tau) \quad g(0) = 1, \quad \lim_{\tau \rightarrow \infty} g = 0$

$$\tau_I \equiv \int_0^\infty g(\tau) d\tau$$

integral time scale

integration by parts $\langle x^2(t) \rangle = 2 \langle v^2 \rangle \int_0^t (t - \tau)g(\tau)d\tau$ where $\int_0^\infty \tau g(\tau)d\tau = O(\tau_I^2) \quad t \gg \tau_I : g = 0$

$t \gg \tau_I$, 1-D : $\langle x^2(t) \rangle = 2D_{tur}t$, 3-D : $\langle x^2(t) \rangle = 6D_{tur}t$, where

$$D_{tur} \equiv \langle v^2 \rangle \tau_I$$
$$\langle v^2 \rangle = (\text{turbulence intensity})^2$$

Rough model for the turbulent transport (analogy with molecular diffusion)

$\langle \mathbf{v}\theta \rangle \approx -D_{tur} \nabla \langle \theta \rangle, \qquad \langle \nabla \cdot (\mathbf{v}\theta) \rangle \approx -D_{tur} \Delta \langle \theta \rangle$

limited to scalar mixing with small displacement / size (blobs, sheets ..) $l_I \ll L \quad \left(v_I \approx \langle v^2 \rangle^{1/2}, \quad l_I \equiv v_I \tau_I \right)$
turbulence intensity
integral length scale

$$D_{tur} = l_I v_I$$

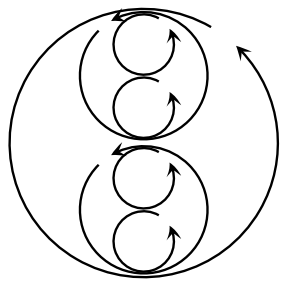
Well-stirred flame regime of Damköhler (1940) $l_I \ll d_L$ and $D_{tur} \gg D_T$

little practical importance

$U_{tur} \approx \sqrt{D_{tur}/\tau_b} \qquad \frac{U_{tur}}{U_L} \approx \sqrt{\left(\frac{l_I}{d_L}\right) \left(\frac{v_I}{U_L}\right)} \gg 1, \qquad d_{tur} \approx D_{tur}/U_{tur} \gg d_L$

IX-3) Strongly corrugated flammelets regime

Kolmogorov's cascade (homogeneous, isotropic and fully-developed turbulence)



schematic representation
 $l_i = l_I / 2^i$

Decomposition into a sum of vortices of different length and time scales

$$l_i, \quad \tau_i, \quad v_i \equiv l_i / \tau_i \quad \text{turn-over velocity} \quad \text{Re}_i \equiv l_i v_i / \nu \quad \text{local Reynolds nb} \quad \nu \equiv \mu / \rho \quad \text{viscous diffusion coeff}$$

$$\text{Kolmogorov scale} \quad l_K, \tau_K, v_K \quad \text{Re}_K = 1 \quad l_i > l_K \quad v_i > v_K \quad \forall i$$

$$\text{Integral scale} \quad l_I, \tau_I, v_I \quad \text{Re}_I \gg 1 \quad l_I > l_i \quad v_I > v_i \quad \forall i$$



Kolmogorov 1941

Scaling laws (dimensional analysis) $l_K \ll l_i \ll l_I$

$$\text{energy transfer in NS eqs : } \rho(\mathbf{v} \cdot \nabla) v^2 / 2 \quad v_i^3 / l_i \equiv \epsilon \approx \text{cst} \Rightarrow v_i \approx \epsilon^{1/3} l_i^{1/3}, \quad v_i^2 \approx \epsilon^{2/3} l_i^{2/3}, \quad \tau_i \approx \epsilon^{-1/3} l_i^{2/3},$$

$$\text{dissipation rate of energy : } \nu \mathbf{v} \cdot \Delta \mathbf{v} \Rightarrow \epsilon = \nu v_K^2 / l_K^2 \quad \epsilon = v_I^3 / l_I$$

$$\text{Re}_K \equiv v_K l_K / \nu = 1 \Rightarrow l_I / l_K \approx \text{Re}_I^{3/4}, \quad v_I / v_K \approx \text{Re}_I^{1/4}, \quad \tau_I / \tau_K \approx \text{Re}_I^{1/2} \quad \text{Re}_I \gg 1$$

$$\text{energy spectrum : } \langle v^2 \rangle / 2 = \int_0^\infty dk E(k) \quad E(k) \approx \epsilon^{2/3} k^{-5/3}$$

definition of strongly corrugated flames

$$v_K \ll U_L \ll v_I \Rightarrow d_L \ll l_K, \quad \tau_L \ll \tau_K \quad \text{no modification to the laminar flame structure}$$

$$v_K \approx \nu / l_K, \quad U_L \approx D_T / d_L, \quad D_T \approx \nu$$

Gibson scale l_G (Peters 1986)

definition of the Gibson scale: smallest size of the wrinkles on the flame front

$$\text{turn-over time} = \text{transit time across the vortex} \quad \tau_i \approx l_i / U_L \Rightarrow v_i \approx U_L$$

$$l_G \equiv U_L^3 / \epsilon \Rightarrow l_K \ll l_G \ll l_I$$

many scales of wrinkles $l_G \ll l_I \Rightarrow$ fractal geometry of the flame front

Elements of fractal geometry

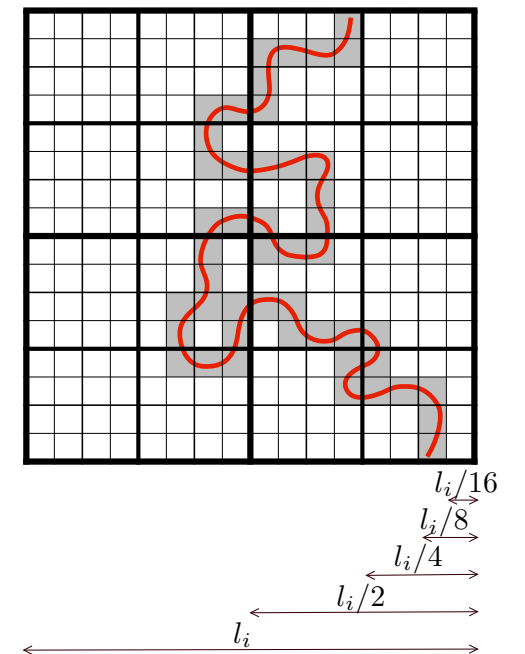
Total surface area in a cube of size l_i , $l_G < l_i < l_I$ $S_i \approx N_{i,G} l_G^2$.
 nb of cubes of size l_G that intersect the surface within the volume l_i^3 $\nearrow$

Weaker resolution l_j , $l_G < l_j < l_i$ $S_{i,j} \approx N_{i,j} l_j^2$
 nb of cubes of size l_j that intersect the surface within the volume l_i^3 $\nearrow$

Details of small scales are lost as the size of the box l_j increases

$$S_{i,j+k} < S_{i,j} < S_i \quad N_{i,j+k} < N_{i,j} < N_i$$

Fractal dimension $D_f > 2$: $N_{i,j} \approx (l_i/l_j)^{D_f}$, $S_{i,j}/l_i^2 \approx (l_i/l_j)^{D_f-2}$



Regular surface: $D_f = 2 \Rightarrow$ total area S_i in a box of size l_i $S_i/l_i^2 = \lim_{l_j \rightarrow 0} S_{i,j}/l_i^2 = \text{finite cst.}$

For a flame of thickness d_L its area is well defined for wrinkles whose scale is larger than d_L , $l_j > d_L$

The fractal dimension $D_f > 2$ can concern only scales greater than the smallest wrinkles

Fractal dimension of a turbulent flame can be meaningful only for $d_L < l_G < l_j < l_i < l_I$

Assumption: the Kolmogorov cascade is not modified by gas expansion ok for $l_i \gg l_G$

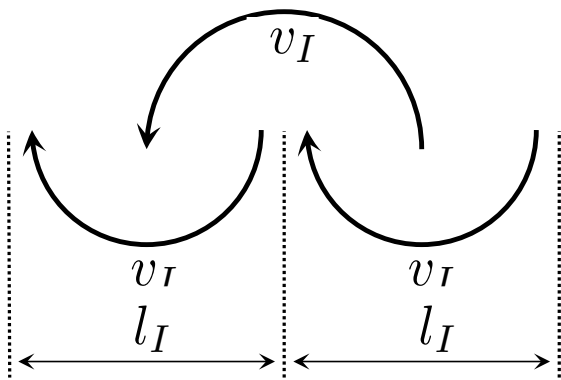
Contamination time vs combustion time

Kolmogorov cascade

$\tau_i \approx \epsilon^{-1/3} l_i^{2/3} \searrow l_i \searrow$
 $v_i \approx \epsilon^{1/3} l_i^{1/3} \nearrow l_i \nearrow$

Fastest contamination: integral scale $v_I \gg v_i$. $U_{tur} = v_I$?

ok if the combustion time of the vortex is not longer than the turnover time



Self similar law

An effective front of thickness l_i is defined at each scale

A flame velocity U_i can be defined at each scale if $U_i = (S_{i,j}/l_i^2)U_j$ $U_i/U_j = \langle S_{i,j} \rangle / l_i^2$

At the Gibson scale the combustion time of the vortex = turnover time $U_L = v_G$

Self similarity: same law at all scales $\Rightarrow$ combustion time of the vortex = the turnover time $\forall l_i$

$l_i/U_i = \tau_i \quad \Rightarrow \quad U_i = v_i$

Kolmogorov cascade $\Rightarrow$ small vortices burn faster than larger ones

$U_{tur} = v_I, \quad l_{tur} = l_I$

Fractal dimension of the flame surface:

$U_i/U_j = \langle S_{i,j} \rangle / l_i^2 \Rightarrow u_i/u_j = \langle S_{i,j} \rangle / l_i^2 \Rightarrow \langle S_{i,j} \rangle / l_i^2 = (l_i/l_j)^{1/3} \Rightarrow$
 $\langle S_{i,j} \rangle / l_i^2 = (l_i/l_j)^{D_f-2}$

$D_f = 7/3$

The result is the same for all mixtures...??

Co-variant laws

Pocheau 1994

More general law independent of the turbulent scaling and satisfying additivity

Turbulent energy contained in the range $[l_i, l_j]$: $v_{i,j}^2 \equiv \sum_{k=i}^{j-1} v_k^2$ v_k^2 : energy in $[k, k+1]$

Co-variant law = same for each couple of length scales l_i, l_j $l_i > l_j$

The only co-variant law for the flame velocity U_i at scale l_i satisfying additivity is $U_i^2 = U_j^2 + c v_{i,j}^2$

Co-variance ? $l_i > l_k > l_j$, $v_{i,j}^2 = v_{i,k}^2 + v_{k,j}^2$ $U_i^2 = U_j^2 + c v_{i,k}^2 + c v_{k,j}^2 = U_k^2 + c v_{i,k}^2$ Pocheau 1994

co-variance ok $U_i^2 = U_k^2 + c v_{i,k}^2$

$$U_{tur}^2 = U_L^2 + c v^2$$

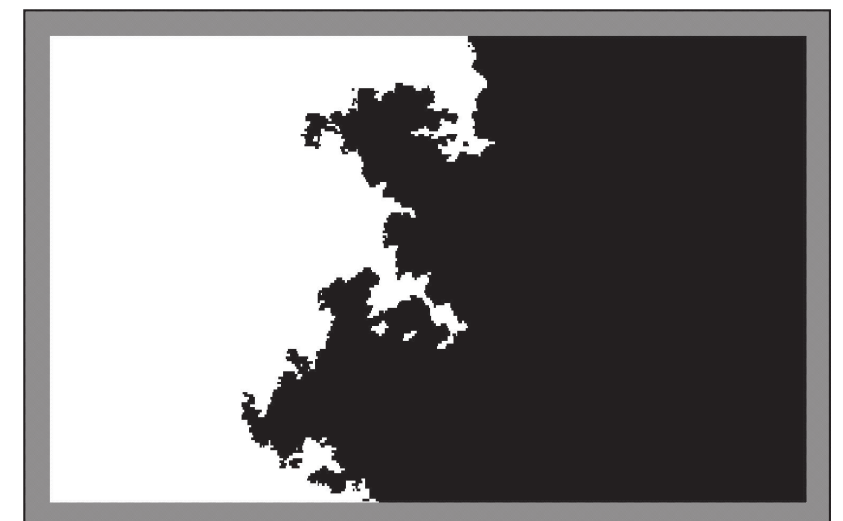
$v^2 \equiv \sum_{\forall n} v_n^2$ turbulence intensity

Not limited to a strong turbulence

The case $c = 1$ covers the known results at low and large turbulence intensity

Reasonably good agreement with experiments

$$v/U_L = O(1), \quad l_I/l_K \approx 180$$



Pocheau 1996

IX-4) Turbulent combustion noise

wavelength $a/\omega \gg L$ size of the flame

Monopolar sound emission

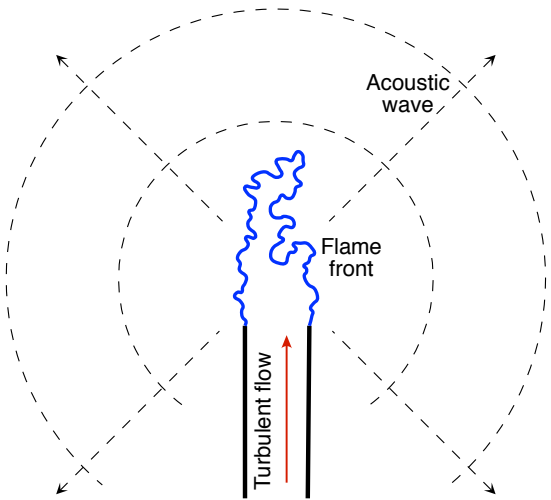
Deformable (small) body with fluctuating volume $V(t)$

$\mathbf{u} = \nabla \phi(\mathbf{r}, t)$ acoustic potential $\phi(\mathbf{r}, t) = -\frac{\dot{V}(t - ar)}{4\pi r}$ $r \equiv |\mathbf{r}|$, $\dot{V}(t) \equiv dV/dt$

$r \gg L : v = (4\pi ar)^{-1} \ddot{V}(t - r/a), \quad \ddot{V}(t) \equiv d^2V(t)/dt^2$

Radiated flux of energy (intensity of sound) $I \equiv \rho a \langle v^2 \rangle$

$$I = (\rho/4\pi a) \left\langle \left(d^2V/dt^2 \right)^2 \right\rangle$$



Sound generated by a turbulent flame $dV/dt = \dot{M}_b/\rho_b$

$\dot{M}_b = \rho_b \iint_S (\mathcal{D}_f + U_b) d^2\sigma$ $\dot{M}_u = \rho_u \iint_S (\mathcal{D}_f + U_L) d^2\sigma$ $\rho_u U_L = \rho_b U_b$

Annotations:
 - $\mathcal{D}_f$: normal flame velocity in the lab frame
 - $\dot{M}_b$: mass flow rate of burned gas in the lab frame
 - $\dot{M}_u$: mass flow rate of fresh gas in the lab frame

elimination of $\mathcal{D}_f$ $dV/dt = \dot{M}_b/\rho_b = \dot{M}_u/\rho_u + (U_b - U_L)S$ $dV/dt = (U_b - U_L)S(t)$

$\dot{M}_u/\rho_u$ is labeled as **constant** with a red arrow.

$$I = (\rho/4\pi a)(U_b - U_L)^2 \langle (dS/dt)^2 \rangle$$

intensity of sound

Strahle 1985

$$d\tilde{I}(\omega)/d\omega = \frac{\rho}{4\pi a}(U_b - U_L)^2 \int_0^\infty dt e^{i\omega t} \langle \dot{S}(t)\dot{S}(0) \rangle$$

power spectrum of sound

intensity of sound

$$I = (\rho/4\pi a)(U_b - U_L)^2 \langle (dS/dt)^2 \rangle$$

power spectrum of sound

$$d\tilde{I}(\omega)/d\omega = \frac{\rho}{4\pi a}(U_b - U_L)^2 \int_0^\infty dt e^{i\omega t} \langle \dot{S}(t)\dot{S}(0) \rangle$$

Stongly corrugated regime with a Kolmogorov cascade:

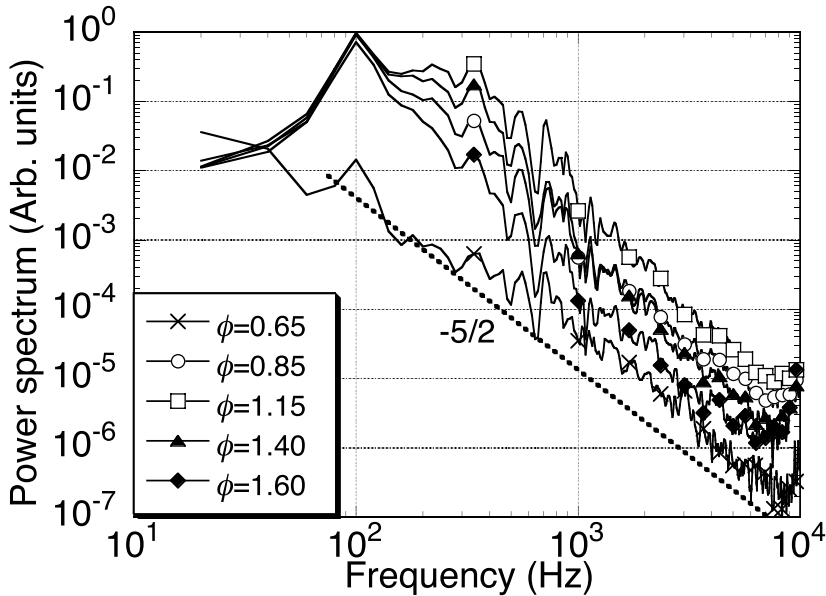
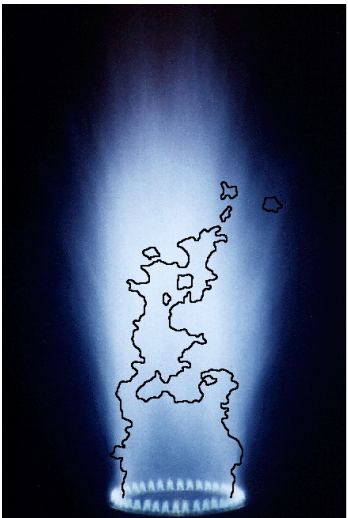
$$D_f = 4/3 \quad \Rightarrow \quad I \approx \frac{1}{4\pi} \left(\frac{T_b}{T_u} - 1 \right)^2 (\rho \Delta V) \frac{v_I^4}{al_I}$$

total volume of the flame brush

$$d\tilde{I}(\omega) \propto \omega^{-5/2} d\omega$$

Clavin Siggia 1991

in agreement with experiments on very large burners
(Abugov Obrezkov 1978)



Blowtorch noise

Combustion noise is two orders of magnitude higher

the noise is not resulting from the direct interaction of upstream turbulence on the flame front
amplification by the intrinsic flame instability is essential

(Searby 2001)

