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Structure and Dynamics of Combustion Waves in Premixed Gases

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Lecture IV
Hydrodynamic instability of flames

Lecture 4 : Hydrodynamic instability of flames

4-1. Jump across an hydrodynamic discontinuity

4-2. Linearized Euler equations of an incompressible fluid

4-3. Conditions at the front

4-4. Dynamics of passive interfaces

4-5. Darrieus-Landau instability

4-6. Curvature effect: a simplified approach

$IV-1)$ Jump across an hydrodynamic discontinuity

$\boxed{\text{flame considered as a discontinuity} \quad \text{flame thickness and curvature neglected}}$

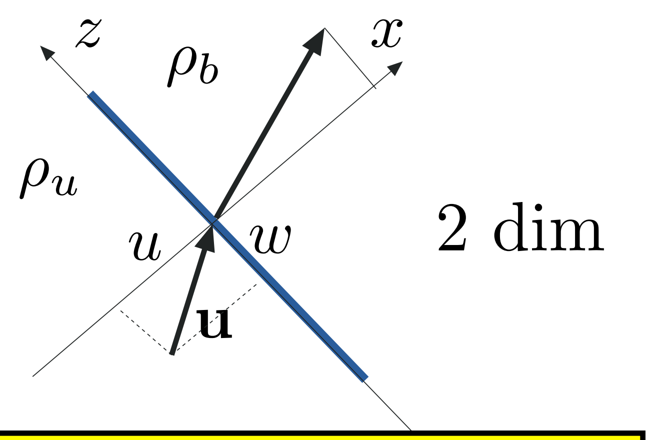
$\Lambda \gg d_L$

flame $\approx$ surface of zero thickness separating two incompressible flows

Low Mach nb approx + inviscid approx: Euler eqs

$$\partial\rho/\partial t = -\nabla.(\rho\mathbf{u}) \qquad \rho \mathrm{D}\mathbf{u}/\mathrm{D}t = -\nabla p \;\Leftrightarrow\; \partial(\rho\mathbf{u})/\partial t = -\nabla.(p\underline{\underline{\mathbf{I}}} + \rho\underline{\underline{\mathbf{uu}}})$$

tilted planar front



2 dim

reference frame of the flame $\qquad \mathbf{r} = (x, z), \quad \mathbf{u} = (u, w)$

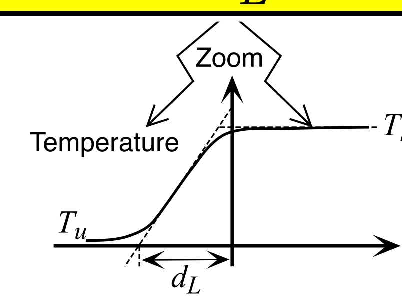
$$\partial\rho/\partial t = -\partial(\rho u)/\partial x - \partial(\rho w)/\partial z,$$

$$\partial(\rho u)/\partial t = -\partial(p + \rho u^2)/\partial x - \partial(\rho u w)/\partial z,$$

$$\partial(\rho w)/\partial t = -\partial(\rho u w)/\partial x - \partial(p + \rho w^2)/\partial z$$

$$\boxed{\lim_{d_L \to 0} \int_{d_L} a(x, y, t)\mathrm{d}x = 0}$$

if $a(\mathbf{r}, t)$ is regular



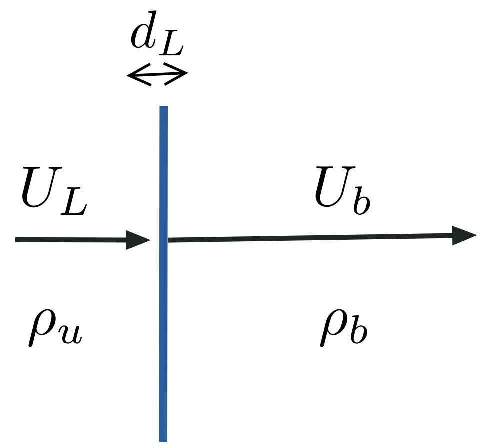
jump relations (reference frame of the flame)

$$[\rho u]_-^+ = 0 \qquad \rho u = \rho_u U_L = \rho_b U_b$$

$$[p + \rho u^2]_-^+ = 0$$

$$\rho u \neq 0 \quad \Rightarrow \quad [w]_-^+ = 0$$

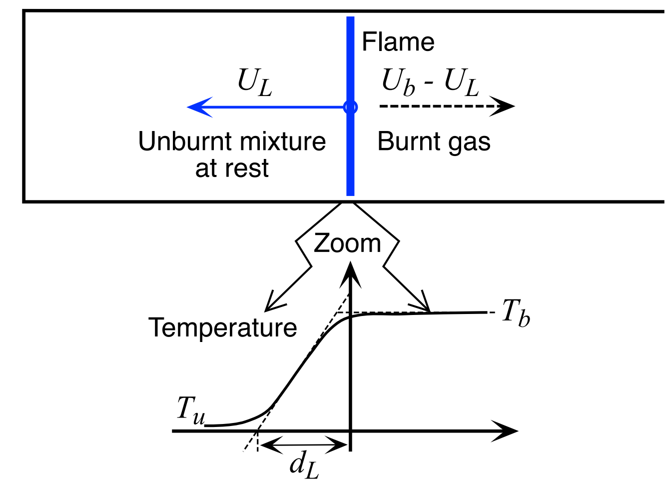
$$\boxed{\rho_u > \rho_b}$$



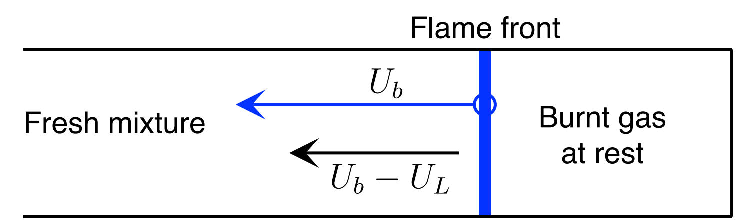
reference frame of the flame front

$$\frac{U_b}{U_L} = \frac{\rho_u}{\rho_b} = \frac{T_b}{T_u}$$

conservation of mass + isobaric approx

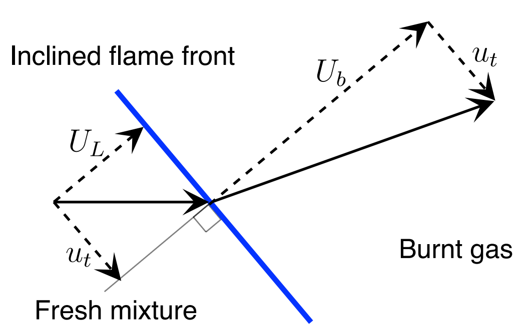


Piston effect

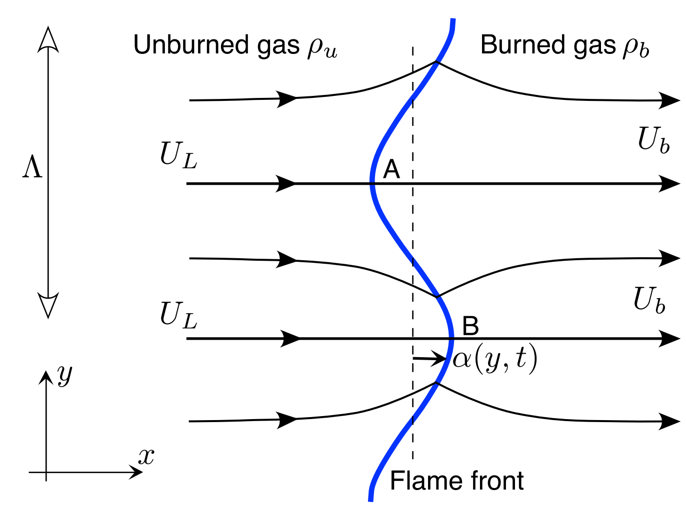


"instantaneous" modification of the flow field, both upstream and donstream
(low Mach nb approx: the speed of sound is infinite, $a \approx \infty$)

tilted front



deviation of the stream lines



$u_-|_A < U_L$
flame motion

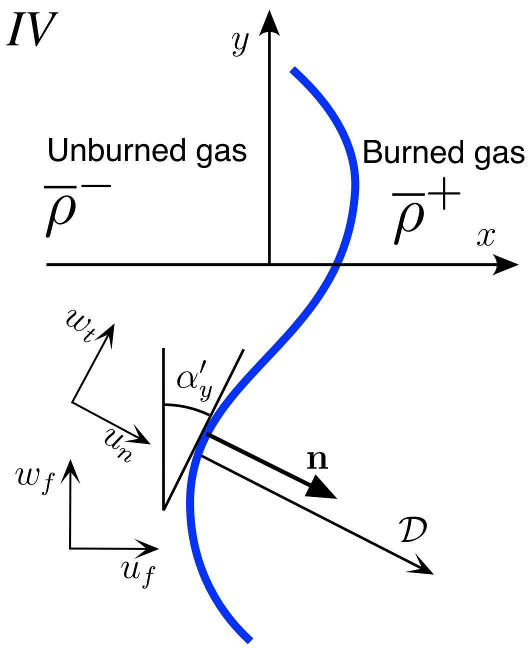
$u_-|_B > U_L$
flame motion

instability mechanism

$$\boxed{\sigma \propto U_L/\Lambda}$$

$$\Lambda \gg d_L, \quad d_L/\Lambda \to 0$$

P.Clavin IV



equation of the perturbed front $\quad x = \alpha(y,t)$

(reference frame !)

flow velocity at the front $\quad \mathbf{u}_f = (u_f, w_f)$

$$\mathbf{n}_f = \left(\frac{1}{\sqrt{1+\alpha_y'^2}}, -\frac{\alpha'_y}{\sqrt{1+\alpha_y'^2}} \right), \quad u_n \equiv \mathbf{u}_f.\mathbf{n}_f = (u_f - \alpha'_y w_f)/\sqrt{1+\alpha_y'^2}$$

$$w_{tg} = (w_f + \alpha'_y u_f)/\sqrt{1+\alpha_y'^2}$$

$$\alpha'_y \equiv \partial\alpha/\partial y$$

$$\dot{\alpha}_t \equiv \partial\alpha/\partial t$$

normal velocity of the front

$$\mathcal{D}_f = \frac{\dot{\alpha}_t}{\sqrt{1+\alpha_y'^2}}$$

reference frame of the planar unperturbed flame $(x=0)$

flow velocity relative to the perturbed front

$$U_n \equiv u_n - \mathcal{D}_f = \left(u_f - \dot{\alpha}_t - \alpha'_y w_f\right)/\sqrt{1+\alpha_y'^2} \qquad W_{tg} = w_{tg}$$

normal component $\qquad$ tangentail component

conservation of mass

$$\overline{\rho}^- U_n^- = \overline{\rho}^+ U_n^+$$

$$\boxed{\overline{\rho}^- \left(u_f^- - \dot{\alpha}_t - \alpha'_y w_f^-\right) = \overline{\rho}^+ \left(u_f^+ - \dot{\alpha}_t - \alpha'_y w_f^+\right)}$$

conservation of momentum

$$[p + \overline{\rho} U_n^2]_-^+ = 0 \qquad [W_{tg}]_-^+ = 0$$

$$\boxed{p_f^- + \overline{\rho}^- \frac{\left(u_f^- - \dot{\alpha}_t - \alpha'_y w_f^-\right)^2}{1+\alpha_y'^2} = p_f^+ + \overline{\rho}^+ \frac{\left(u_f^+ - \dot{\alpha}_t - \alpha'_y w_f^+\right)^2}{1+\alpha_y'^2} \qquad \left(w_f^- + \alpha'_y u_f^-\right) = \left(w_f^+ + \alpha'_y u_f^+\right)}$$

$IV-2)$ Linearised Euler equations of an incompressible fluid

$$\frac{\partial}{\partial x}\delta u^{\pm}+\frac{\partial}{\partial \mathbf{y}}\delta \mathbf{w}^{\pm}=0,$$

$$a=\overline{a}+\delta a$$
$$\overline{m}_f=\overline{\rho}^-\overline{u}_f^-=\overline{\rho}^+\overline{u}_f^+$$

$$\left(\overline{\rho}^{\pm}\frac{\partial}{\partial t}+\overline{m}_f\frac{\partial}{\partial x}\right)\delta u^{\pm}=-\frac{\partial}{\partial x}\delta\pi^{\pm} \qquad \pi^{\pm}\equiv p^{\pm}-\overline{\rho}^{\pm}g(t)x,$$

$$\left(\overline{\rho}^{\pm}\frac{\partial}{\partial t}+\overline{m}_f\frac{\partial}{\partial x}\right)\delta \mathbf{w}^{\pm}=-\frac{\partial}{\partial \mathbf{y}}\delta\pi^{\pm},$$

$x\longrightarrow+\infty$: disturbances remain finite,

$x\longrightarrow-\infty$: no disturbances, $\quad \delta u^-=0$

$$\boxed{\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\delta\pi^{\pm}=0}$$

transverse coordinates

$$\boxed{\delta a(x,\mathbf{y},t)=\tilde{a}(x,t)\mathrm{e}^{\mathrm{i}\mathbf{k}.\mathbf{y}} \qquad \alpha(\mathbf{y},t)=\tilde{\alpha}(t)\mathrm{e}^{\mathrm{i}\mathbf{k}.\mathbf{y}}}$$

pressure $\quad \dfrac{\partial^2\tilde{\pi}^{\pm}}{\partial x^2}-|\mathbf{k}|^2\tilde{\pi}^{\pm}=0 \qquad \boxed{\tilde{\pi}^{\pm}(x,t)=\tilde{\pi}_f^{\pm}(t)\mathrm{e}^{\mp|\mathbf{k}|x}}$

flow velocity $\quad \dfrac{\partial\tilde{u}^{\pm}}{\partial x}+\mathrm{i}\mathbf{k}.\tilde{\mathbf{w}}^{\pm}=0 \qquad \overline{\rho}^{\pm}\left(\dfrac{\partial}{\partial t}+\overline{u}^{\pm}\dfrac{\partial}{\partial x}\right)\tilde{u}^{\pm}(x,t)=\pm|\mathbf{k}|\tilde{\pi}_f^{\pm}(t)\mathrm{e}^{\mp|\mathbf{k}|x}$

general solution to the homogeneous equation + particular solution

$$\boxed{\tilde{u}^{\pm}(x,t)=\tilde{u}_R^{\pm}(x,t)+\tilde{u}_P^{\pm}(x,t)}$$

$$\tilde{u}_P^{\pm}(x,t)=\tilde{u}_p^{\pm}(t)\mathrm{e}^{\mp kx},$$

$$\frac{\partial\tilde{u}_R^{\pm}}{\partial t}+\overline{u}^{\pm}\frac{\partial\tilde{u}_R^{\pm}}{\partial x}=0, \qquad \overline{\rho}^{\pm}\left(\frac{\mathrm{d}}{\mathrm{d}t}\mp\overline{u}^{\pm}k\right)\tilde{u}_p^{\pm}(t)=\pm k\tilde{\pi}_f^{\pm}(t)$$

$$\boxed{\tilde{u}^{\pm}(x,t) = \tilde{u}_R^{\pm}(x,t) + \tilde{u}_P^{\pm}(x,t)}$$

$$k \equiv |\mathbf{k}| = 2\pi/\Lambda \qquad \tilde{u}^- = \tilde{u}_P^- = \tilde{u}_f^-(t)\mathrm{e}^{kx} \quad \tilde{u}_P^+ = \tilde{u}_p^+(t)\mathrm{e}^{-kx}$$

potential flows

$$\frac{\partial \tilde{u}_R^{\pm}}{\partial t} + \overline{u}^{\pm}\frac{\partial \tilde{u}_R^{\pm}}{\partial x} = 0, \qquad \tilde{u}_R^{\pm} = \tilde{u}_r^{\pm}(t - x/\overline{u}^{\pm}), \qquad \tilde{u}_R^- = 0, \quad \tilde{u}_R^+ = \tilde{u}_r^+(t - x/\overline{u}^{\pm}),$$

vorticity of the burnt gas flow

3 unknown functions: $\tilde{u}_f^-(t)$, $\tilde{u}_p^+(t)$, $\tilde{u}_r^+(t)$

$$x < 0: \quad \begin{cases} \tilde{u}^-(x,t) = \tilde{u}_f^-(t)\mathrm{e}^{+kx}, \\ k\tilde{\pi}^-(x,t) = -\overline{\rho}^-\left(\dfrac{\mathrm{d}}{\mathrm{d}t} + \overline{u}^- k\right)\tilde{u}_f^-(t)\mathrm{e}^{+kx}, \end{cases}$$

$$x > 0: \quad \begin{cases} \tilde{u}^+(x,t) = \tilde{u}_p^+(t)\mathrm{e}^{-kx} + \tilde{u}_r^+(t - x/\overline{u}^+), \\ k\tilde{\pi}^+(x,t) = \overline{\rho}^+\left(\dfrac{\mathrm{d}}{\mathrm{d}t} - \overline{u}^+ k\right)\tilde{u}_p^+(t)\mathrm{e}^{-kx}, \end{cases}$$

$$\mathrm{i}\mathbf{k}.\tilde{\mathbf{w}}^-(x,t) = -k\tilde{u}^-(x,t), \quad \mathrm{i}\mathbf{k}.\tilde{\mathbf{w}}^+(x,t) = -\frac{\partial}{\partial x}\tilde{u}^+(x,t).$$

4 boundary conditions at the flame front involving the additional unknown $\tilde{\alpha}(t)$

2 for the conservation of mass (inner flame structure not modified)

$$\boxed{\delta m_f^- = \delta m_f^+ = 0} \qquad m \equiv \rho(u - \partial\alpha/\partial t)$$

2 for the conservation of normal and tangential momentum

IV-3) Conditions at the front

notation $a(x,t)$

$a_f(t) \equiv a(x=0,t)$

transverse coordinates

$\delta a(x,\mathbf{y},t) = \tilde{a}(x,t)\mathrm{e}^{\mathrm{i}\mathbf{k}.\mathbf{y}} \qquad \alpha(\mathbf{y},t) = \tilde{\alpha}(t)\mathrm{e}^{\mathrm{i}\mathbf{k}.\mathbf{y}}$

Mass

$$\boxed{\overline{\rho}^-\left(\delta u_f^- - \dot{\alpha}_t\right) = \overline{\rho}^+\left(\delta u_f^+ - \dot{\alpha}_t\right) = 0} \Rightarrow \quad \delta u_f^- = \delta u_f^+ = \dot{\alpha}_t \qquad \tilde{u}_f^-(t) = \tilde{u}_p^+(t) + \tilde{u}_r^+(t) = \mathrm{d}\tilde{\alpha}/\mathrm{d}t$$

$$\Downarrow$$

$$\frac{\mathrm{d}\tilde{u}_r^+}{\mathrm{d}t} = -\frac{\mathrm{d}\tilde{u}_p^+}{\mathrm{d}t} + \frac{\mathrm{d}^2\tilde{\alpha}}{\mathrm{d}t^2}$$

Tangential momentum

$$\boxed{\frac{\partial}{\partial y}\left(w_f^- + \alpha'_y\overline{u}^-\right) = \frac{\partial}{\partial y}\left(w_f^+ + \alpha'_y\overline{u}^+\right)} \Rightarrow k\tilde{u}_p^+(t) + \frac{1}{\overline{u}^+}\frac{\mathrm{d}\tilde{u}_r^+(t)}{\mathrm{d}t} + k\tilde{u}_f^-(t) = \overline{m}_f\left(\frac{1}{\overline{\rho}^+} - \frac{1}{\overline{\rho}^-}\right)k^2\tilde{\alpha}(t)$$

$$\mathrm{i}\mathbf{k}.\tilde{\mathbf{w}}^-(x,t) = -k\tilde{u}^-(x,t), \quad \mathrm{i}\mathbf{k}.\tilde{\mathbf{w}}^+(x,t) = -\frac{\partial}{\partial x}\tilde{u}^+(x,t).$$

$$\partial\alpha'_y/\partial y \to -k^2\tilde{\alpha}(t)$$

elimination of $\mathrm{d}\tilde{u}_r^+/\mathrm{d}t$

$$\Downarrow$$

$$\boxed{\left(\frac{1}{\overline{u}^+}\frac{\mathrm{d}}{\mathrm{d}t} - k\right)\tilde{u}_p^+ = -\overline{m}_f\left(\frac{1}{\overline{\rho}^+} - \frac{1}{\overline{\rho}^-}\right)k^2\tilde{\alpha} + \frac{1}{\overline{u}^+}\frac{\mathrm{d}^2\tilde{\alpha}}{\mathrm{d}t^2} + \underbrace{k\tilde{u}_f^-}_{k\frac{\mathrm{d}\tilde{\alpha}}{\mathrm{d}t}}}$$

Normal momentum

$$\boxed{\delta p_f^- + 2\overline{\rho}^-\overline{u}_f^-(\cancel{\delta u_f^- - \dot{\alpha}_t}) = \delta p_f^+ + 2\overline{\rho}^+\overline{u}_f^+(\cancel{\delta u_f^+ - \dot{\alpha}_t})} \Rightarrow$$

$$\pi^\pm \equiv p^\pm - \overline{\rho}^\pm g(t)x,$$

$$\tilde{\pi}_f^+ - \tilde{\pi}_f^- = \underbrace{(\overline{\rho}^- - \overline{\rho}^+)g(t)\tilde{\alpha}(t)}_{\text{hydrostatic presure}}$$

$$\Downarrow$$

$$x<0: \begin{cases} \tilde{u}^-(x,t) = \tilde{u}_f^-(t)\mathrm{e}^{+kx}, \\ k\tilde{\pi}^-(x,t) = -\overline{\rho}^-\left(\frac{\mathrm{d}}{\mathrm{d}t} + \overline{u}^-k\right)\tilde{u}_f^-(t)\mathrm{e}^{+kx}, \end{cases}$$

$$x>0: \begin{cases} \tilde{u}^+(x,t) = \tilde{u}_p^+(t)\mathrm{e}^{-kx} + \tilde{u}_r^+(t - x/\overline{u}^+), \\ k\tilde{\pi}^+(x,t) = \overline{\rho}^+\left(\frac{\mathrm{d}}{\mathrm{d}t} - \overline{u}^+k\right)\tilde{u}_p^+(t)\mathrm{e}^{-kx}, \end{cases} \Rightarrow$$

$$\boxed{\overline{m}_f\left(\frac{1}{\overline{u}^+}\frac{\mathrm{d}}{\mathrm{d}t} - k\right)\tilde{u}_p^+ + \overline{m}_f\left(\frac{1}{\overline{u}^-}\frac{\mathrm{d}}{\mathrm{d}t} + k\right)\underbrace{\tilde{u}_f^-}_{\frac{\mathrm{d}\tilde{\alpha}}{\mathrm{d}t}} = (\overline{\rho}^- - \overline{\rho}^+)kg(t)\tilde{\alpha}(t)}$$

elimination of $\tilde{u}_p^+$

$$\Downarrow$$

Equation for the front

$$\tilde{u}_f^- = \mathrm{d}\tilde{\alpha}/\mathrm{d}t \quad \Rightarrow \quad \boxed{(\overline{\rho}^- + \overline{\rho}^+)\frac{\mathrm{d}^2\tilde{\alpha}}{\mathrm{d}t^2} + 2\overline{m}_f k\frac{\mathrm{d}\tilde{\alpha}}{\mathrm{d}t} - \left[(\overline{\rho}^- - \overline{\rho}^+)g(t)k + (\overline{u}^+ - \overline{u}^-)\overline{m}_f k^2\right]\tilde{\alpha} = 0}$$

IV-4) Dynamics of a passive interface


Rayleigh

$$\overline{m}_f = 0$$

$$(\overline{\rho}^- + \overline{\rho}^+)\frac{\mathrm{d}^2\tilde{\alpha}}{\mathrm{d}t^2} + \cancel{2\overline{m}_f k\frac{\mathrm{d}\tilde{\alpha}}{\mathrm{d}t}} - k[(\overline{\rho}^- - \overline{\rho}^+)g(t) + (\overline{u}^+ \cancel{- \overline{u}^-)\overline{m}_f} k]\tilde{\alpha} = 0$$

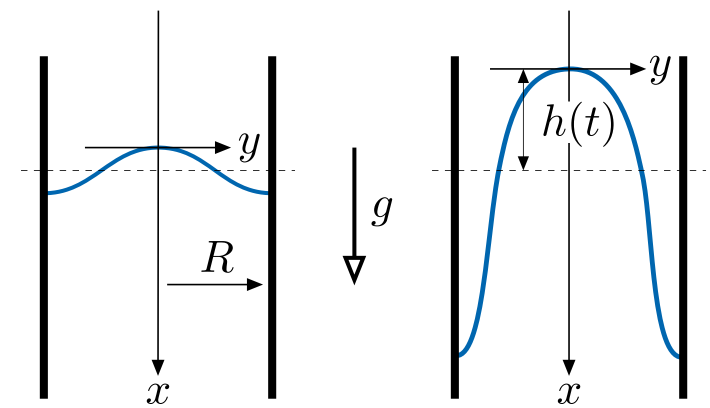
Fourier mode $\boxed{\tilde{\alpha}(t) = \hat{\alpha}\,\mathrm{e}^{\sigma t}}$ $\alpha(\mathbf{y},t) = \tilde{\alpha}(t)\mathrm{e}^{\mathrm{i}\mathbf{k}.\mathbf{y}}$

$\mathrm{Re}(\sigma) < 0:$ linearly stable $\mathrm{Re}(\sigma) > 0:$ linearly unstable

Rayleigh-Taylor instability

$g = \mathrm{cst.}$ $(\overline{\rho}^- - \overline{\rho}^+)g > 0$

$$\sigma^2 - A_{\mathrm{t}} k g = 0, \quad A_{\mathrm{t}} > 0$$



Rayleigh-Taylor bubble (upwards propagation)

$$g > 0, \qquad \mathrm{A_t} \equiv \frac{\rho_- - \rho_+}{\rho_- + \rho_+} > 0 \qquad \sigma = \sqrt{A_{\mathrm{t}} g k} \qquad U_{bubble} = 0.361\sqrt{2gR}$$

ok with dimension

Gravity waves

$g = \mathrm{cst.}$ $(\overline{\rho}^- - \overline{\rho}^+)g < 0$

$$\varpi \equiv \mathrm{Im}\,\sigma \neq 0$$

$$\tilde{\alpha}(t) = \hat{\alpha}\,\mathrm{e}^{\mathrm{i}\varpi t} \qquad \varpi = B\sqrt{gk} \qquad B \equiv \sqrt{\frac{(\rho_+ - \rho_-)}{(\rho_+ + \rho_-)}}$$

Faraday (parametric) instability. Mathieu's equation

$g(t)$ oscillating

$$\frac{\mathrm{d}^2\tilde{\alpha}}{\mathrm{d}t^2} + \varpi_o^2\,[1 + \epsilon\cos(\varpi\tau)]\,\tilde{\alpha} = 0$$

IV-5) Darrieus-Landau instability of flames

Landau

Darrieus

Darrieus 1938
Landau 1944

$$g = 0 \qquad \overline{m}_f = \overline{\rho}^- \overline{u}^- = \overline{\rho}^+ \overline{u}^+$$

$$(\overline{\rho}^- + \overline{\rho}^+)\frac{\mathrm{d}^2\tilde{\alpha}}{\mathrm{d}t^2} + 2\overline{m}_f k \frac{\mathrm{d}\tilde{\alpha}}{\mathrm{d}t} - (\overline{u}^+ - \overline{u}^-)\overline{m}_f k^2 \tilde{\alpha} = 0$$

$$\boxed{\tilde{\alpha}(t) = \hat{\alpha}\, \mathrm{e}^{\sigma t}}$$

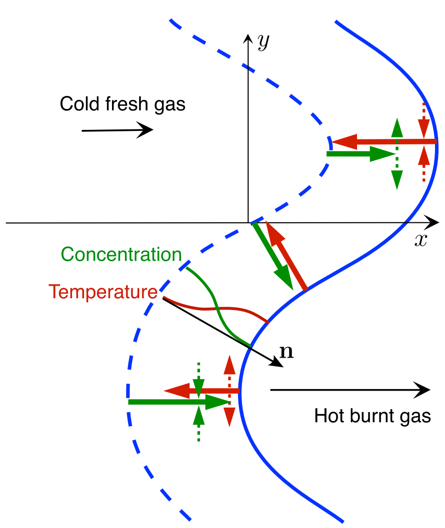
$$\upsilon_b \equiv \overline{\rho}^- / \overline{\rho}^+ = \overline{u}^+ / \overline{u}^- > 1 \qquad (\overline{\rho}^- + \overline{\rho}^+)\sigma^2 + 2\overline{m}_f k\sigma - (\overline{u}^+ - \overline{u}^-)\overline{m}_f k^2 = 0$$

$$\overline{u}_- \equiv U_L \qquad \frac{\sigma}{U_L k} = \frac{1}{1+\upsilon_b^{-1}}\left[-1 \pm \sqrt{1 + \upsilon_b - \upsilon_b^{-1}}\right] \qquad \boxed{\sigma = \mathcal{A} U_L k, \quad \mathcal{A} > 0}$$

$d_L/\Lambda \to 0$: no length scale in the problem; dimensional analysis $\Rightarrow \ \sigma \propto U_L k$

$$\rho_u \gg \rho_b : \sigma = \sqrt{U_b U_L}\, k \qquad (\rho_u - \rho_b)/\rho_u \ll 1 : \ \sigma = (U_b - U_L)k/2$$

$k = 2\pi/\Lambda$ shorter is the wavelength stronger is the instability !?
however the analysis is valid only in the limit $d_L/\Lambda \to 0$

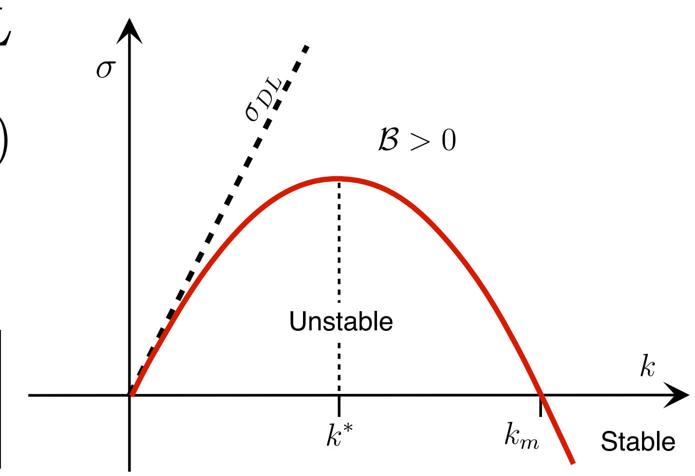


Stabilisation at small wavelength, $\Lambda \approx d_L$

$$\frac{\partial \alpha}{\partial t} = \mathcal{B} D_T \frac{\partial^2 \alpha}{\partial y^2}. \qquad \sigma_{diff} \equiv 1/\tau_{diff} = -\mathcal{B} D_T k^2 = -\mathcal{B} U_L k (d_L k)$$

first order correction $\quad \mathcal{B} > 0\,?$

$$k d_L < 1 : \quad \boxed{\sigma / U_L = \mathcal{A}k - \mathcal{B}k^2 d_L + \ldots}$$



VI-6) Curvature effect: a simplified approach

modification to the inner flame structure $\delta m_f^- \overset{?}{=} \delta m_f^+ \neq 0$

first order in perturbation analysis $d_L/\Lambda \ll 1$ $\quad \delta m_f^-/\overline{\rho}^- \equiv (\delta u_f^- - \dot{\alpha}_t) = -\mathcal{B} D_T \partial^2\alpha/\partial y^2$

$$\tilde{m}_f^-(t)/\overline{m}_f \approx \mathcal{B} d_L k^2 \tilde{\alpha}(t) \qquad D_T = U_L d_L$$

Normal momentum $\quad \delta p_f^- + 2\overline{\rho}^- \overline{u}_f^- (\delta u_f^- - \dot{\alpha}_t) = \delta p_f^+ + 2\overline{\rho}^+ \overline{u}_f^+ (\delta u_f^+ - \dot{\alpha}_t)$

(flame notations: $\overline{\rho}^+ \to \rho_b,\ \ \overline{\rho}^- \to \rho_u,\ \ \rho_u > \rho_b$)

$$\tilde{\pi}_{f+} - \tilde{\pi}_{f-} = -2\overline{m}_f \left(\frac{1}{\rho_b} - \frac{1}{\rho_u}\right) \tilde{m}_f(t) + (\rho_u - \rho_b) g(t) \tilde{\alpha}(t)$$

equation for the flame front (correction due to curvature, finite thickness effect $kd_L \neq 0$)

$$(\rho_u + \rho_b)\frac{\mathrm{d}^2\tilde{\alpha}}{\mathrm{d}t^2} + 2\overline{m}_f k \frac{\mathrm{d}\tilde{\alpha}}{\mathrm{d}t}\,(1 + \cancel{\mathcal{B}kd_L}) = k\tilde{\alpha}(\rho_u - \rho_b)\left[g(t) + U_b U_L k\,(1 - 2\mathcal{B}kd_L)\right]$$

curvature effect

flame propagating downwards $g < 0$

$$\frac{1}{k_m} \equiv 2\mathcal{B} d_L \qquad \boxed{\left(1 + \frac{\rho_b}{\rho_u}\right)\frac{\mathrm{d}^2\tilde{\alpha}}{\mathrm{d}t^2} + 2U_L k \frac{\mathrm{d}\tilde{\alpha}}{\mathrm{d}t} = \left(\frac{\rho_u}{\rho_b} - 1\right) k \left[-\frac{\rho_b}{\rho_u}|g| + U_L^2 k \left(1 - \frac{k}{k_m}\right)\right] \tilde{\alpha}}$$

non-dimensional parameters $\quad \upsilon_b \equiv \overline{\rho}^-/\overline{\rho}^+ = \overline{u}^+/\overline{u}^- > 1$

$$s = \sigma \tau_L \qquad \kappa \equiv k d_L \qquad \kappa_m \equiv 1/(2\mathcal{B}) \qquad \mathcal{G}_0 \equiv (\rho_b/\rho_u)\mathrm{Fr}^{-1} \quad \mathrm{Fr}^{-1} \equiv |g| d_L/U_L^2$$

$$(1 + \upsilon_b^{-1}) s^2 + 2\kappa s - (\upsilon_b - 1)\kappa \left[-\mathcal{G}_o + \kappa\left(1 - \frac{\kappa}{\kappa_m}\right)\right] = 0$$

Stability limits of flames propagating downwards $\quad \sigma = 0$

marginal wavenumber

$$\boxed{\left[-\mathcal{G}_o + \kappa\left(1 - \frac{\kappa}{\kappa_m}\right)\right] = 0,}$$

Stability limits of flames propagating downwards

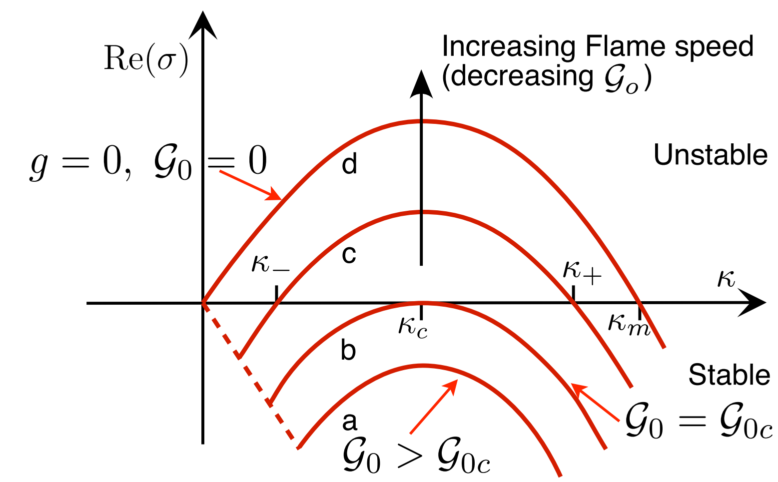
non-dimensional parameters $\kappa \equiv k d_L$ $\quad \kappa_m \equiv 1/(2\mathcal{B})$ $\quad \mathcal{G}_0 \equiv (\rho_b/\rho_u)\mathrm{Fr}^{-1}$ $\quad \mathrm{Fr}^{-1} \equiv |g| d_L / U_L^2$

$s = \sigma \tau_L$

$$(1+v_b^{-1})s^2 + 2\kappa s - (v_b - 1)\kappa\left[-\mathcal{G}_o + \kappa\left(1 - \frac{\kappa}{\kappa_m}\right)\right] = 0$$

marginal wavenumber $\sigma = 0$

$$\boxed{\left[-\mathcal{G}_o + \kappa\left(1 - \frac{\kappa}{\kappa_m}\right)\right] = 0,}$$



gravity stabilizes the large wavelengths of slow propagating flame

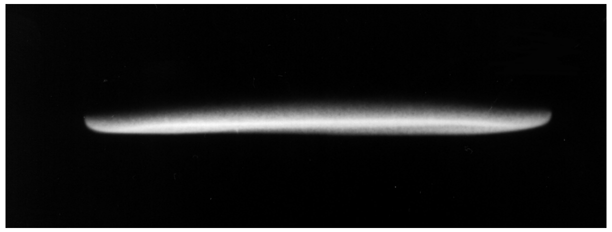
curvature stabilizes the small wavelengths

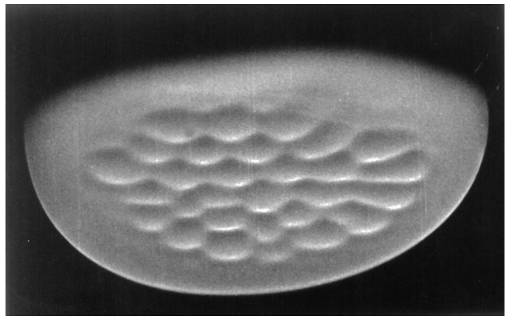
gravity stabilizes the large wavelengths of slow propagating flame $U_L < 10\mathrm{cm/s}$

instability threshold $U_L \approx 10\mathrm{cm/s}$

$$\mathcal{G}_{oc} = \frac{k_c d_L}{2}, \quad k_c = \frac{k_m}{2}, \quad U_{Lc} = \sqrt{2\frac{\rho_b}{\rho_u}\frac{|g|}{k_c}}$$

OK with experiments by Boyer Quinard and Searby (1982)





Flames propagating upwards: bubble flames